

Evaluation of Smoothing Methods for Estimating Mortality Dynamics over Age and Time with Application to US Mortality

Kirill Andreev

Queen's University, Canada. E-mail: andreevk@post.queensu.ca

Abstract

Temporal mortality dynamics is usually analyzed by exploring trends in age-specific death rates or by calculating aggregate measures such as life expectancy at birth. Deeper insights into mortality changes can be gained by exploring surfaces of mortality improvement over age and time. Statistical methods for estimating such surfaces are underdeveloped as this topic emerged only recently in demographic and actuarial research with accumulation of reliable data on human mortality spanning over extended periods of time. In this work two smoothing methods, iterated kernel smoother and tensor-product spline, are tested on simulated data to evaluate their performance in revealing typical patterns of mortality improvement. Finally, the methods are applied for estimating surface of US mortality improvement over period 1933–2000.

Objective

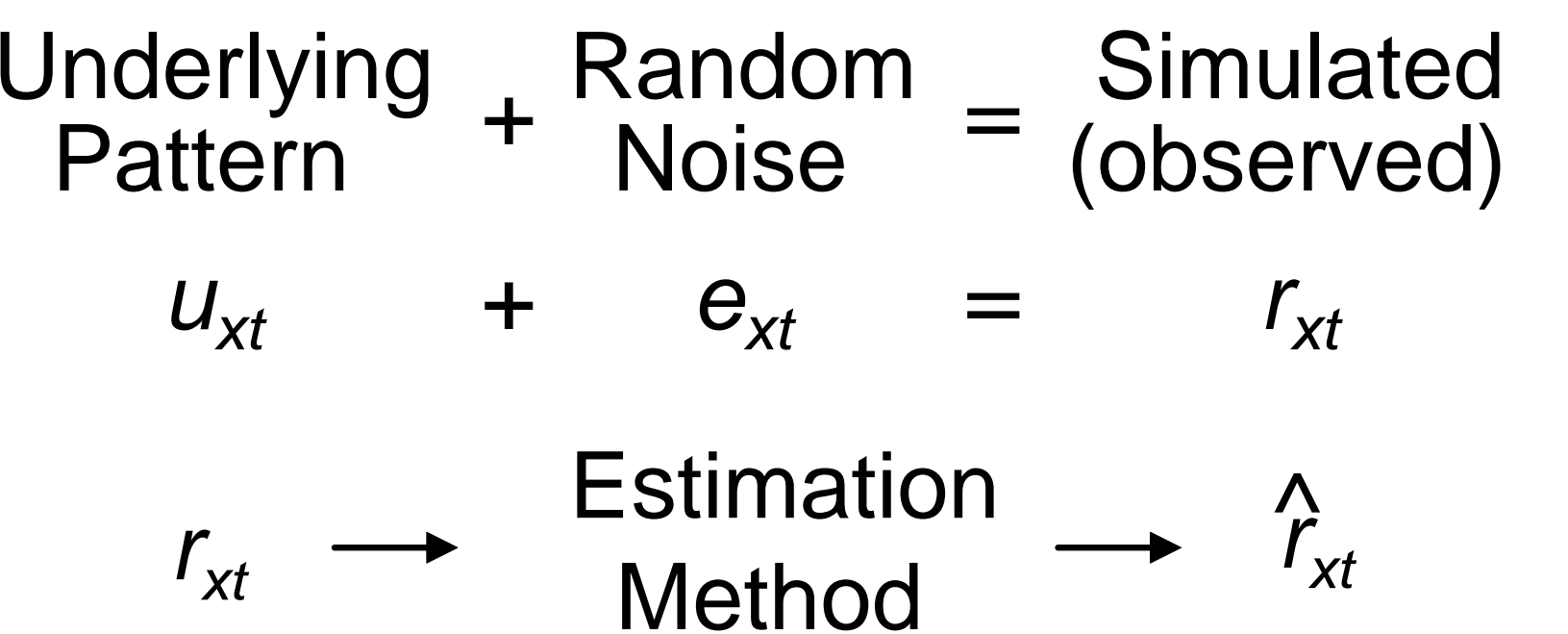
Develop an approach for estimating surface of mortality improvement over age and time.

Simulations

Patterns of improvement, u_{xt}

- A) Constant rate of decline
- B) Age shifting pattern
- C) Lee-Carter mortality dynamics

Scheme



Methods

- Tensor Product Spline (TPS). Degree of smoothing is regulated by the smoothing factor $\lambda=[0,1]$;
- Iterated Kernel Smoother with Epanechnikov (IKSe) and uniform kernels (IKSu). Degree of smoothing is regulated by the number of iterations, n .

Goodness of estimation

$$\text{Integrated square error} \quad ISE = \sum (\hat{r}_{xt} - u_{xt})^2$$

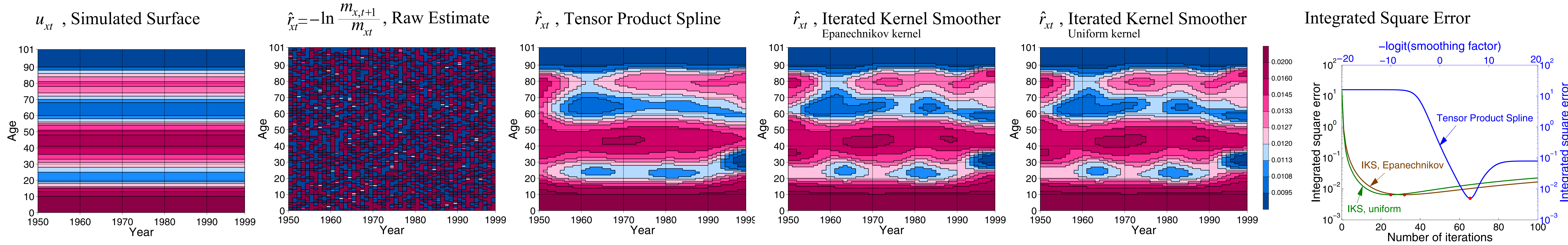
Results & Conclusions

- approximation of underlying surface of mortality improvement was found to be adequate;
 - optimal values of parameters are about $\lambda = 5e-3$ for TPS and $n = 25..30$ for IKS;
 - performance of the TPS was only marginally better than that of the IKS;
 - Minimal Integrated Square Error
- | SIM* | IKSe | IKSu | TPS |
|------|---------|----------------|----------------|
| A) | 6.59e-3 | 6.48e-3 | 4.96e-3 |
| B) | 9.68e-3 | 9.66e-3 | 6.94e-3 |
| C) | 1.44e-1 | 1.43e-1 | 1.61e-1 |
- * — Simulation/Method
- data analysis can benefit from exploration of mortality improvement surfaces computed at different degrees of smoothing, preferably in a form of computer-assisted animation;
 - application of smoothing methods to the US female mortality revealed that progress made against mortality at ages above 60 in the last two decades was unusually moderate contrary to findings in other developed countries.

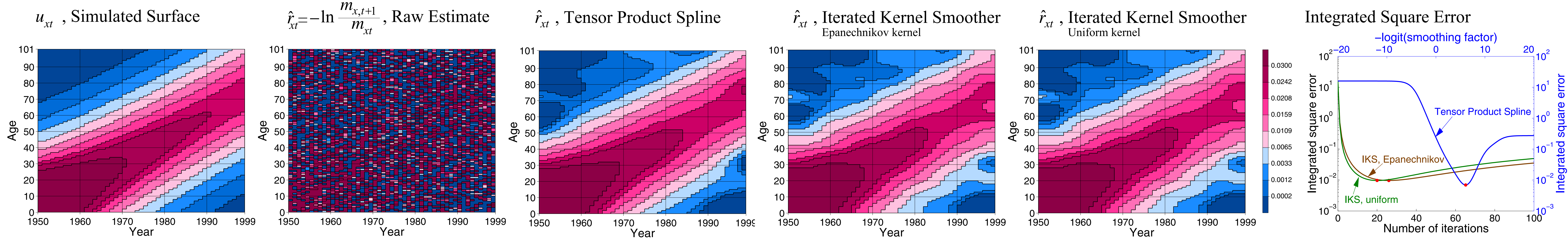
Acknowledgments

This project was supported by grant R01 AG18444 from the U.S. National Institute on Aging.

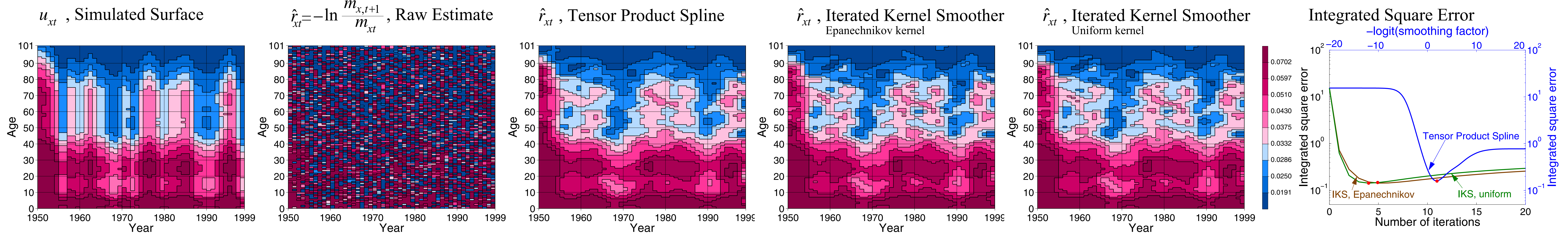
Constant Rate of Decline



Age Shifting Pattern of Mortality Decline



Lee-Carter Mortality Dynamics



United States, Females: Surface of Mortality Improvement over Age and Time Estimated by Tensor Product Spline at Different Levels of the Smoothing Factor

