Age Patterns of Mortality Improvement by Level of Life Expectancy at Birth with Applications to Mortality Projections

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Abstract

Mortality projections are often produced by a two-step procedure: first, projections of life expectancy at birth are prepared and, second, life tables consistent with the projected levels of life expectancy at birth are constructed. We propose a new method to construct projected life tables incorporating typical age-specific patterns of mortality improvement. The patterns have been estimated from mortality dynamics of individual countries included in the Human Mortality Database. Such approach is important as it aids more accurate projections of future age patterns of human mortality.

Disclaimer

Introduction

Any method designed for projection of mortality has to produce a period life table for each of the projection periods. The period life table incorporates complete information about mortality experience for the projection period including life expectancy at birth (or e0; in demographic or actuarial notation). One of the commonly used approaches for computing life tables is to perform calculations in two steps: first, to make projections of life expectancy at birth and, second, to construct a life table consistent with the projected levels. The approach is often used if mortality projections have to be produced simultaneously for a large number of countries as future life expectancy trends must appear plausible not only for a particular country but also between countries. Constructing life tables consistent with the life expectancy projections is an additional challenge as there is no unique way to select such a life table: life tables with drastically different age patterns of mortality can be characterized by the same levels of life expectancy at birth. From infinite number of life tables consistent with the projected level of life expectancy at birth, a more plausible choice seem to be a life table satisfying to two general conditions: 1) age pattern of mortality in the current period is close to the age pattern in the previous one and 2) transition from one mortality pattern to another is consistent with transitions observed in other countries with similar levels of life expectancy at birth. The first condition can be satisfied by incorporating information about age pattern of mortality from the previous projection period for producing age pattern of mortality for the current one. The second condition will be satisfied if age pattern of mortality improvement implied by mortality transition from one period to the next will be, by and large, consistent with age patterns of mortality improvement experienced by other countries as they progress from one level of life expectancy to another. This idea is developed in the subsequent sections of this article.

Data and Methods

Suppose that mortality in the period $[t_1, t_1 + 5]$ is represented by an abridged life table with life expectancy at birth equal to $e_0(t_1)$ and age-specific schedule of death rates $m_x(t_1)$. Assume also that over period $\Delta t = t_2 - t_1$ life expectancy at birth is expected to reach level $e_0(t_2)$ with the corresponding schedule of death rates $m_x(t_2)$. Transition from one schedule of death rates $m_x(t_1)$ to another $m_x(t_2)$ can be expressed as

$$\ln m_x(t_2) = \ln m_x(t_1) - k(t_1)\rho_x(t_1) \tag{1}$$

where $\rho_x(t_1)$ is normalized ($\sum \rho_x = 1$) age-specific pattern of mortality improvement and $k(t_1)$ is a coefficient governing the level of mortality reduction over period Δt.

As average rate of progress in reducing mortality rates at age *x* is equal to

$$\ln m_x(t_1) - \ln m_x(t_2) \cong \left(1 - \frac{m_x(t_2)}{m_x(t_1)}\right) \tag{2}$$

The quantity *k* can be also interpreted as an overall rate of progress in reducing mortality rates obtained by summing age-specific rates of progress:

$$k \cdot 100\% = 100\% \cdot \sum_x \left(\ln m_x(t_1) - \ln m_x(t_2)\right) \cong \sum_x \left(1 - \frac{m_x(t_2)}{m_x(t_1)}\right) \cdot 100\% \tag{3}$$

We estimated patterns of mortality improvement ρ_x using abridged life tables for 5-year periods from Human Mortality Database (HMD). Our estimation procedure is illustrated on estimating ρ_x for females and for levels of life expectancy at birth 65-70 years. First step was to select life tables from the life table dataset with life expectancy at birth equal approximately 65 years (we used two years tolerance and selected life tables with e0 in a range 63-67 years). Similarly, we selected another set of life tables with e0 equal approximately 70 years, all life tables with e0 in a range 68-72 years. Next step was to select two life tables, one from the first set and another from the second set, and to compute pattern of mortality improvement observed for the current pair *i* of life tables:

$$\rho_{xi} = -c_i(\ln m_{x2} - \ln m_{x1})$$

where c_i is a normalizing constant chosen to satisfy $\sum \rho_x = 1$.

As all available empirical patterns of mortality improvement ρ_{xi} had been computed, we found the 50th percentile and smoothed it with a cubic spline (smoothing parameter = 8e-2) (Fig. 1). Light lilac paths in Figure 1 are the empirical patterns of mortality improvement, ρ_{xi}, the blue curve is the 50th percentile and the red curve is the fitted cubic spline, out final estimate. For computing ρ_{xi} we limited ourselves to transitions for the same country and sex only that significantly reduces empirical dataset available for the estimation. More empirical data can be obtained by including transitions between countries on expense that many of them are not genuine mortality transitions. Patterns of mortality improvement for other 5-year increments of life expectancy at birth have been estimated in a similar way.

[Figure 1]

Available demographic data permits reliable estimation of the patterns of mortality improvement up to 75-80 years of e0 for males, and 80-85 years for females. For extrapolating patterns of mortality improvement into higher levels of life expectancy at birth, usually assumed in mortality projections, we fitted ordinary least square regressions to each of the age groups of the ρ_x matrix and used the fitted lines, slightly smoothed over age, to compute ρ_x for levels of life expectancy at birth up to 105-110 years of age (Table 1).

Results

Figure 2 shows patterns of mortality improvement ρ_x estimated for females for selected levels of life expectancy at birth. The three selected curves capture well patterns in Table 1. For lower levels of life expectancy at birth (blue curve, legend item e0=60-65), the values of ρ_x are high for younger ages but decline rapidly with advancing age. For higher levels of e0 (red curve), less progress against mortality is expected ages below 30, for ages 30-60 the pattern stays almost flat until age group 70-75, and then it drops down rapidly revealing progressively slower mortality improvements with advancing age. The last curve (green) shows our extrapolations of mortality improvement pattern into level of life expectancy of 95-100 years of age. Less progress against mortality is expected below age 55 while mortality improvements above 55 become more prominent with definite peak at ages 75-80. After 80, as for the red curve (e0=85-80), the rates of progress are declining but their levels are consistently higher than that of the red curve indicating that more progress against mortality will take place in progressively higher age groups.

[Figure 2]

Changes in mortality improvement patterns over level of life expectancy at birth and over age can be well represented with the Lexis maps. Figure 3 presents the data from Table 1 in compact and revealing manner. Each rectangle in Fig. 3 corresponds to a single cell from Table 1 and its color is chosen according to the linear scale on the right. The scale progressively changes color from the dark blue for the very low levels of mortality improvement to the dark magenta hue for the very high levels. The blue areas in the map represent thus low levels of mortality improvement and the magenta areas show high levels. For levels of life expectancies of 70 or less, progress in reduction of mortality took place mostly at younger ages as depicted by the dark magenta areas in the lower left corner of the maps. Over age, irregularly over level of e0 but more or less consistent with age, rates of progress against mortality become lower as shown by transition from the dark magenta areas at younger ages to light hues, and finally to the blue hues over age.

Starting with life expectancy level of about 70 we observe a structural shift in mortality improvement patterns: less progress against mortality takes place at younger ages while decline at adult and high ages accelerates. Our extrapolations into higher levels of life expectancy at birth show further reductions in rates of progress at younger ages and persistent gains at higher ages: the dark magenta areas at the bottom of the map are shrinking with increasing e0 while the magenta area in the middle of the map, ages 55-95, is expanding with a peak at 75-80. Shrinkage of the blue areas at the top of the map is also an indication that more progress in mortality reduction is expected to take place at the very high ages.

[Figure 3]

Example of computing life tables for the projection period using estimated and extrapolated patterns of mortality improvement

In this section we provide a detail example how our estimated and projected patterns of mortality improvement (Table 1) can be used for mortality projections. Given current life table with age pattern of mortality $m_x(t_1)$ and the target life expectancy at birth $e_0(t_2)$, construction of a life table for the next projection period t_2 requires selection of mortality improvement pattern $\rho_x(t_1)$ and a such level of mortality reduction $k(t_1)$ that the life expectancy at birth corresponding to $m_x(t_2)$ computed with Equ. (1) is equal to $e_0(t_2)$. Computations are numerically simple and we demonstrate them on example of computing a life table for 2010-2015 for female population of Japan.

In 2005-2010, Japanese female life expectancy at birth was 85.95 years and according to the projections prepared by United Nations Population Division (United Nations, 2011) an increase to 87.12 years in 2010-2015 was expected. Abridged life table for 2005-2010 is available from HMD and we included death rates from the life table into column mx(2005), Table 2. First, we selected age pattern of mortality improvement corresponding to e0 in 2005-2010. As e0 was equal to 85.95, we selected pattern of mortality improvement from the column e85-90, Table 1, females. Next step was to find level of mortality reduction *k* consistent with increase in e0 to 87.12 years of age. We did it for setting an objective function which takes a single input parameter *k*, computes a life table for the given value of *k*, and returns difference between e0 corresponding to the current value of k and the target level of 87.12. This objective function can be used as input for any zero-search algorithm to automatically finding the necessary value of *k* (we used fzero() function available in MATLAB). In this example we found that *k* = 3.03 is needed to achieve e0=87.12 in 2010-2015 given the selected pattern of mortality improvement and the mortality schedule in 2005-2010. Age-specific death rates for 2005-2010 are given in column mx(2010), Table 2.

Figure 4a shows observed and projected death rates for 2005-2010 and 2010-2015, respectively.

[Figure 4]

Visually, the difference between two curves is small but it accounts for more than a year in life expectancy at birth: 87.12-85.95 = 1.17. Lower panel, Figure 4b, shows log of ratio of death rates, in 2005-2010 to those in 2010-2015 or, equivalently, scaled pattern of mortality improvement: $\ln[m_x(2005)/m_x(2010)] = k(2005)\rho_{xf}(85)$. Figure 4b can be also interpreted as pattern of relative reductions in death rates between two periods (Equ. 2) with each data point showing approximately a percentage of reduction in death rates by age divided by 100.

One can see that the highest reductions are expected in infancy where 1*m*0 is expected to drop by more than 20%. Over age rate of progress declines reaching minimum of 10% at 45-50 where it starts to increase again reaching maximum of 15% at 75-80. After the peak, it declines sharply to almost zero at 110.

In our example, *k* was estimated to be equal to 3.03 or, in other words, that about 300% of mortality reduction is needed over all ages (Equ. 3) to sustain anticipated increase in life expectancy at birth from 85.95 years of age in 2005-2010 to 87.12 in 2010-2015.

Changes in mortality pattern over longer period of time

To investigate changes in a mortality pattern over a longer period of time implied by the application of the estimated and extrapolated patterns of mortality improvements, we prepared mortality projection of Japanese female population for years 2010-2100 (we will refer to this type of projection as PMI hereafter). The last period with the observed data was 2005-2010 and the last period with the projected data was 2095-2050. We used projections of life expectancy at birth published in 2010 Revision of World Population Prospects (United Nations, 2011) where e0 is expected to increase from 80.6 in 2005-2010 to 90.8 years in 2095-2100. The projection was prepared as described above with patterns of mortality improvements were varying over the projection horizon according to the current level of life expectancy at birth.

For comparison we also prepared a projection by Lee-Carter method (Lee and Carter, 1992; Lee and Miller, 2001) adapted to be constrained to the same trends in life expectancy at birth by selecting appropriate increases in the k_t parameter for each of the projection periods (Andreev and Vaupel, 2006).

Parameters a_x and b_x of the Lee-Carter model were estimated by fitting the model to the Japanese female death rates for 1950-2010 from HMD.

Figure 5 shows observed age pattern of death rates in 2005-2010 and the age patterns produced by PMI (PMI series) and Lee-Carter method (L-C series). Both, L-C and PMI age schedules of mortality have corresponding life expectancy at birth equal to 90.8 years. Compared to L-C death rates, PMI death rates are higher below age 80 and lower above this age. Less progress in reduction of mortality is expected at ages below 80 if projection is prepared by PMI method as compared with Lee-Carter. On the other hand, more progress is expected at ages higher than 80. On contrary, Lee-Carter model achieves the same level of life expectancy by reducing death rates at younger ages while less mortality improvement takes place at advanced ages. Age pattern of mortality produced by Lee-Carter model has a steeper age-specific increase in death rates resulting in concentration of life table deaths in a narrower age interval and more rectangular survival curve as compared with PMI.

[Figure 5]

Relative reductions in death rates by age groups produced by two models are given in Figure 6. The highest reductions are at age 0: in 2095-2100 PMI death rate is 87% lower as compared with 1m0 in 2005-2010 and the death rate of Lee-Carter model is 96% lower. Below age 80, reductions in mortality for PMI model are below 80%, significantly lower than rates of progress of Lee-Carter model which are well above 80%. Above 80, however, the situation is reversed; more progress in reducing death rates is expected in PMI model as compared with Lee-Carter: 64% vs. 59% at age 90, and 44% vs. 34% at age 100. On average for all age groups, death rates are expected to be reduced by 67% for PMI model and by 75% for Lee-Carter model. Generally, in the case of PMI model transformation of the initial schedule of death rates of 2005-2010 is less dramatic, less extremely low levels of mortality are required in childhood and young adult ages and less overall progress in reducing mortality is needed to achieve the target level of life expectancy at birth, 90.8 years. This makes the schedule of death rates produced by applying time-varying patterns of mortality improvement more plausible than that produced by the Lee-Carter model.

[Figure 6]

Application of mortality improvement patterns for back projections of mortality

In many countries quality of data produced by the civil registration systems has been improving over time. Currently, in many countries, mortality estimates can be produced directly from empirical data on population and deaths, with no or minimal adjustments of the estimates. For historical periods, however, no reliable information is available, and, with a certain degree of confidence, only trends in infant or child mortality could be estimated. The PMI procedure described above could be used for reconstruction of life tables for the historical periods (back projection of mortality) only with minor modifications.

A starting life table, in this case, is an empirical life table for the current period. To compute a life table for the previous period, one needs to select a pattern of mortality improvement corresponding to the current life expectancy at birth and find a value of k in Equ. (1) that makes the resulting life table consistent with available mortality information for this period, e.g. under five mortality rate. Currently, model life table systems (Coale and Demeny 1966, 1983; United Nations 1982) or their modern extensions (Murray et al. 2003; Wilmoth et al. 2012) are widely used as a means of reconstructing life tables from limited data. The model life table systems are based on well-known positive correlation between death rates at different ages. In our case, however, background information incorporated in reconstruction of life tables is different; the PMI method uses the current mortality pattern and dynamics of mortality rates estimated from experience of other countries as they progress from one level of life expectancy to another. Whether the new approach will offer significant benefits over existing methods requires additional extensive research which is outside the scope of this article. Nevertheless, a few tests we conducted make it worth outlying this idea here it here.

Discussion

Life expectancy at birth is a key indicator of population health highly correlated with economic development of a country. For this reason, it often given a priority in mortality projections and its projections are prepared separately from the rest of mortality indicators (United Nations, 2011). In this case, life tables consistent with the projections of life expectancy at birth have to be constructed as a second step of a mortality projection. In this paper, we demonstrated that transition from one level of life expectancy at birth to another was associated historically with a specific pattern of mortality improvement: at lower levels rise in life expectancy at birth was driven mostly by progress against mortality in infancy and childhood while at higher levels the growth was fueled by reductions in mortality at adult and advanced ages (Fig. 2). Extrapolation of the observed age patterns of mortality improvement into the higher levels of life expectancy at birth, which are above the currently observed but often assumed to be reached over next decades, suggests that the age pattern of mortality will continue to shift to the advanced ages, and the growth of life expectancy at birth will be driven by mortality reductions at progressively higher ages.

There are several reasons why incorporating the estimated age patterns of mortality improvement into mortality projections can lead to construction of more plausible life tables. In many countries, e.g. Russia, historical trends in life expectancy at birth are very irregular and age-specific trends in death rates are diverging in certain periods. In 1990-2005, for example, Russian death rates at ages 25-65 were increasing while child mortality continued to decline. If a positive outlook for the future is assumed, and life expectancy at birth is anticipated to be continuously increasing, diverging historical trends in age-specific death rates cannot inform future age patterns of mortality: the resultant age patterns will be highly distorted and implausible. On the other hand, by adopting the estimated age patterns of mortality improvements, we explicitly assume that transitions to the higher levels of life expectancy at birth in Russia will proceed similarly to the transitions that occurred in other counters. This will produce more plausible age patterns of future mortality.

Even in the countries where mortality decline was more regular and stable over time (e.g. Japan), and the observed trends in the age-specific death rates can inform construction of the life tables for the projection periods (e.g. by applying Lee-Carter model), adopting the time-varying patterns of mortality improvements, as demonstrated above, produces more plausible age patterns of mortality. Incorporating constant patterns of mortality improvement derived entirely from the historical trends into mortality projections results in extremely low death rates in infancy, childhood and adolescence, often to the extent that their biological plausibility could be questioned. By using instead the mortality improvement patterns estimated here, the resulting age schedules of death rates, as show above, appear more plausible. In addition, overall less dramatic reductions of age-specific death rates are required to achieve the same levels of life expectancy at birth.

Beyond improving construction of life tables for mortality projections, the technique introduced here has many potential applications. For example, it can be used in back projections of mortality for reconstructing mortality and population dynamics for historical periods. Instead of incorporating information about age patterns of human mortality as in the model life table systems, it permits incorporating information about dynamics of death rates over age and time as observed in human populations. In addition, it may be useful as aids in actuarial work as the construction of cohort life tables where projections of future cohort death rates are required for cohorts with incomplete survival history.

Acknowledgments

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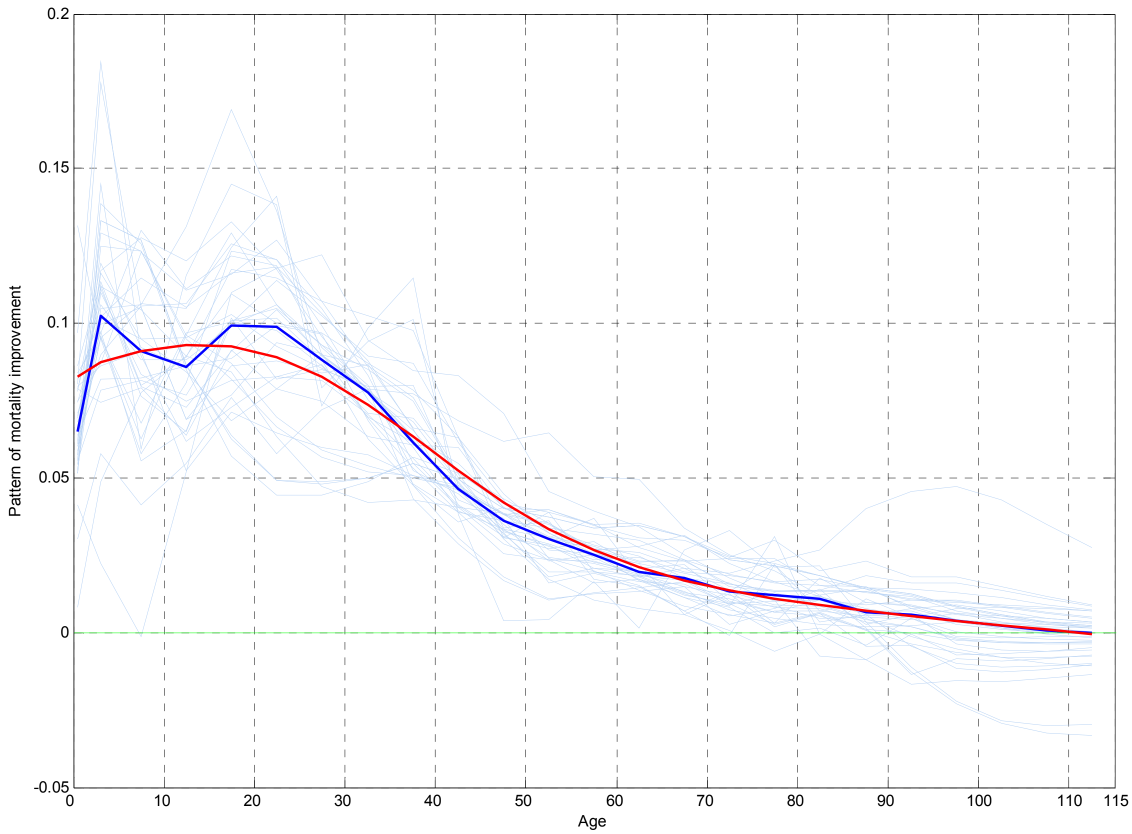
Figure 1 . Example of estimating a pattern of mortality improvement, level of life expectancy at birth 65-70 years of age, females.



Legend
Light lilac curves: empirical patterns of mortality improvement
Blue curve: 50th percentile
Red curve: final estimate, 50th percentile smoothed with a cubic spline (smoothing parameter = 8e-2)
Dataset: 5x5 abridged life tables from HMD, transitions within the same country only

Figure 2 Estimated and extrapolated patterns of mortality improvement for selected levels of life expectancy at birth, females

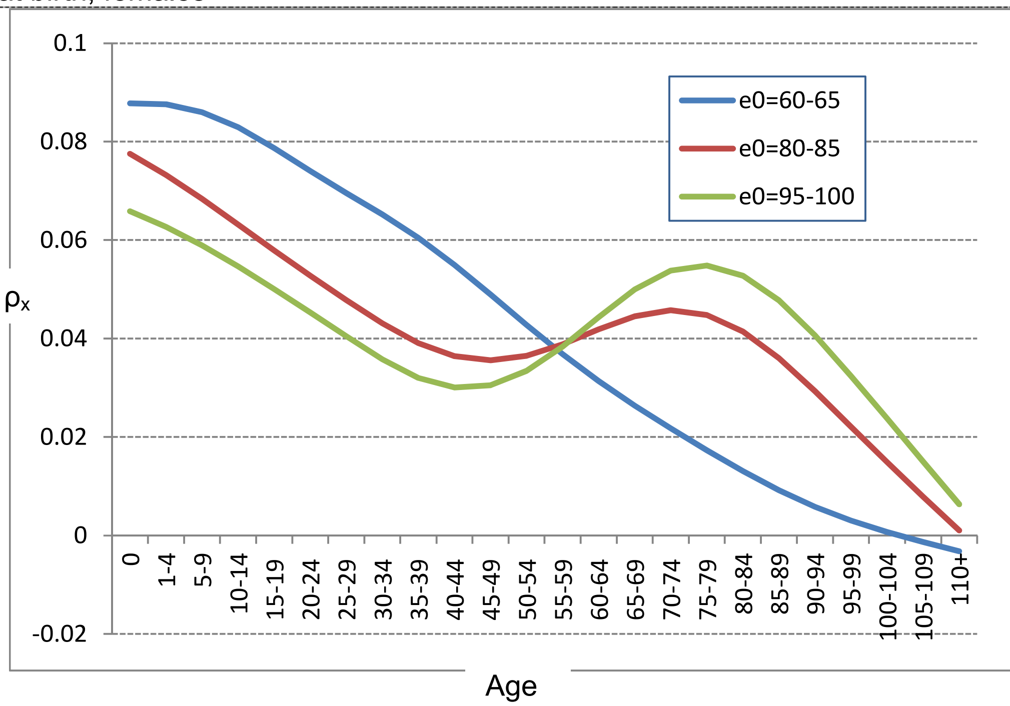


Figure 3. Patterns of mortality improvements by age and by level of life expectancy at birth

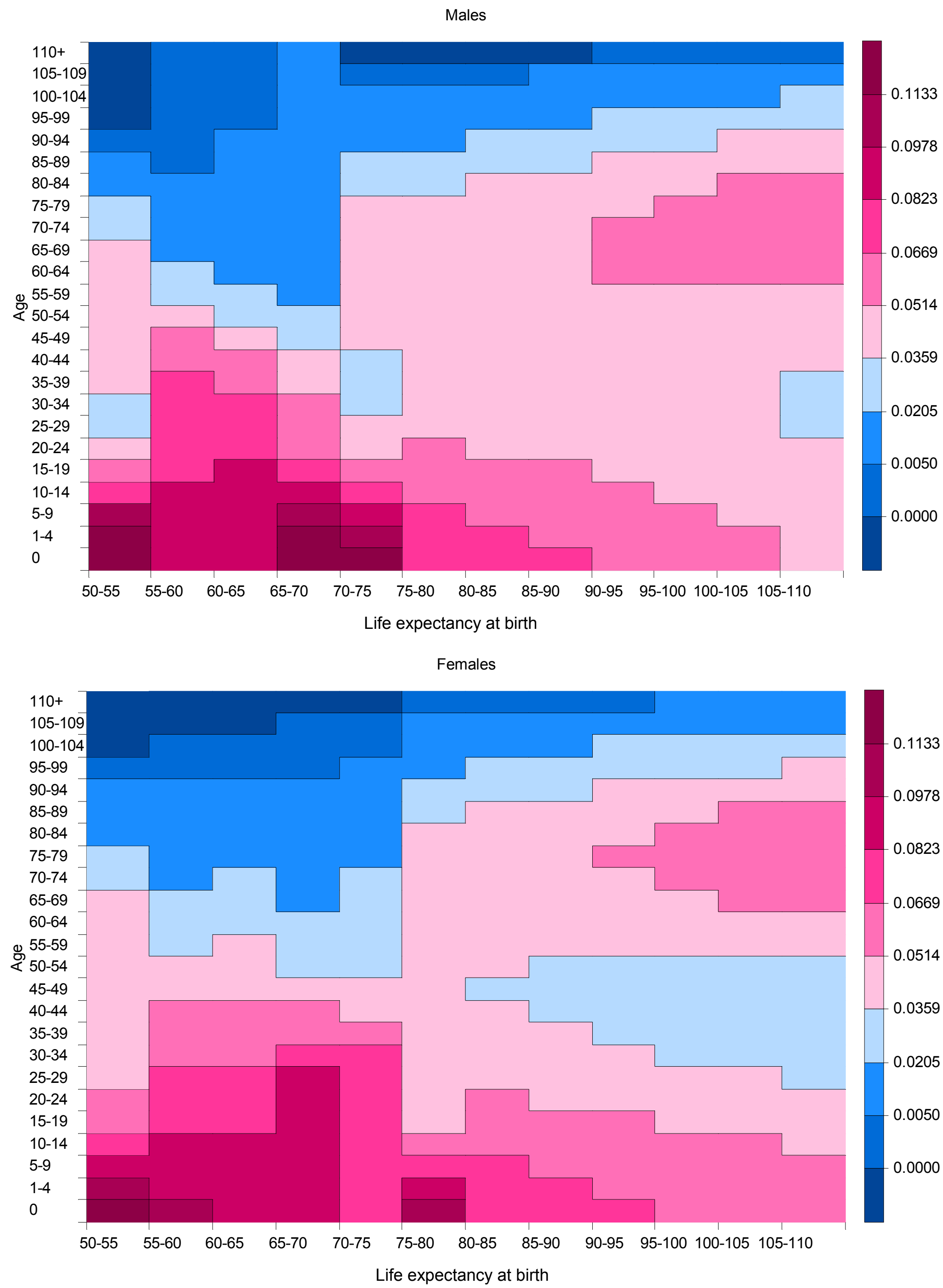


Figure 4a. Japanese female death rates in 2010-2015 projected by applying pattern of mortality improvement for life expectancy level 80-85 to the death rates in 2005-2010

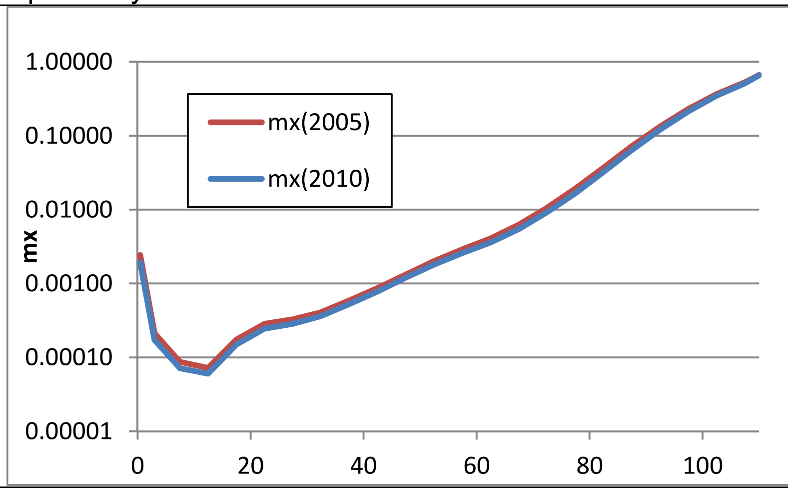
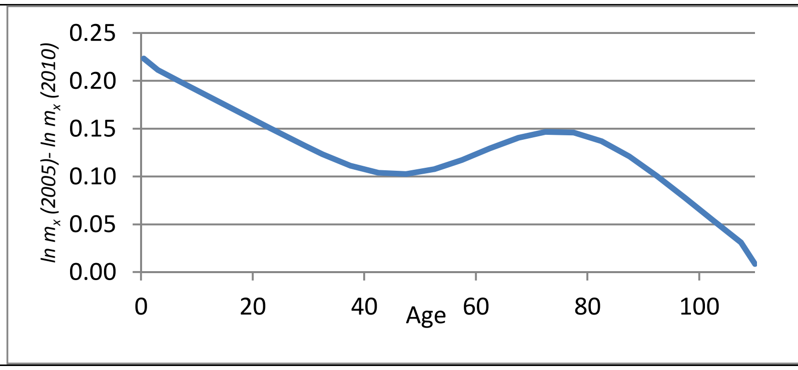
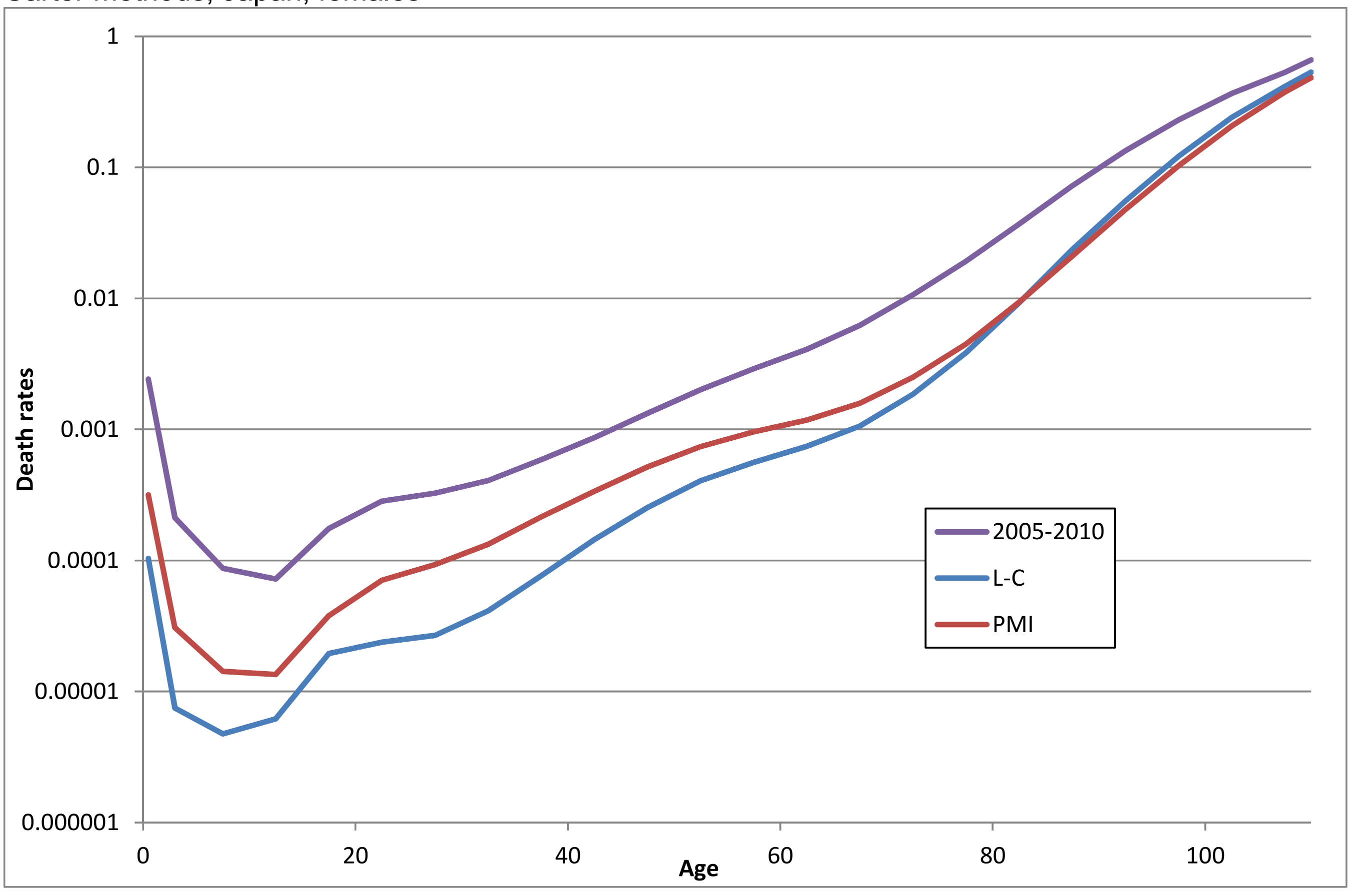



Figure 4b. Log of ratio of death rates in 2005-20010 to those in 2010-2015*


* N.B. the ratio is also equal to the scaled pattern of mortality improvement:

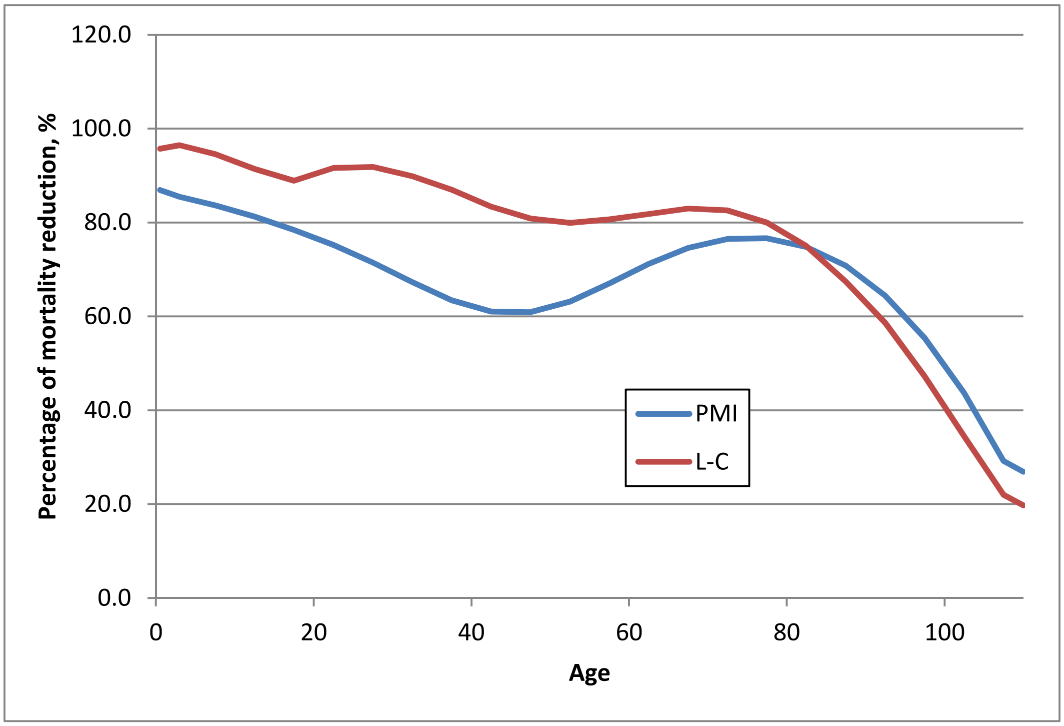
$\ln[m_x(2005)/m_x(2010)] = k(2005)\rho_{xf}(85)$

Figure 5. Changes in age pattern of mortality from 2005-2010 to 2095-2010 produced by PMI and Lee-Carter methods, Japan, females



Legend
2005-2010 – observed death rates in 2005-2010
L-C – death rates in 2095-2100 produced by Lee-Carter method
PMI – death rates in 2095-2100 produced by applying the patterns of mortality improvement (PMI), Table 1.

Figure 6. Reductions in the death rates (%) from 2005-2010 to 2095-2100 produced by PMI and Lee-Carter methods, Japan, females



Legend
L-C – Reductions in death rates implied by PMI method
L-C – Reductions in death rates implied by Lee-Carter method

Table 1 Estimated (black) and extrapolated (blue) patterns of mortality improvements by life expectancy at birth and by sex

Males

age	e50-55	e55-60	e60-65	e65-70	e70-75	e75-80	e80-85	e85-90	e90-95	e95-100	e100-105	e105-110
0	0.1449	0.0826	0.0922	0.1374	0.1151	0.0818	0.0767	0.0715	0.0663	0.0611	0.0559	0.0507
1-4	0.1266	0.0843	0.0912	0.1207	0.1005	0.0757	0.0710	0.0663	0.0616	0.0569	0.0523	0.0476
5-9	0.1055	0.0845	0.0892	0.1036	0.0856	0.0696	0.0655	0.0613	0.0571	0.0530	0.0488	0.0446
10-14	0.0818	0.0833	0.0867	0.0874	0.0702	0.0637	0.0601	0.0565	0.0529	0.0492	0.0456	0.0420
15-19	0.0598	0.0814	0.0844	0.0738	0.0557	0.0582	0.0551	0.0519	0.0488	0.0457	0.0426	0.0394
20-24	0.0438	0.0790	0.0820	0.0642	0.0439	0.0535	0.0508	0.0480	0.0453	0.0426	0.0398	0.0371
25-29	0.0356	0.0762	0.0787	0.0575	0.0360	0.0504	0.0479	0.0455	0.0431	0.0406	0.0382	0.0358
30-34	0.0342	0.0727	0.0734	0.0519	0.0322	0.0487	0.0465	0.0442	0.0420	0.0398	0.0376	0.0354
35-39	0.0375	0.0678	0.0657	0.0460	0.0322	0.0476	0.0456	0.0436	0.0417	0.0397	0.0377	0.0357
40-44	0.0429	0.0610	0.0561	0.0394	0.0346	0.0468	0.0451	0.0435	0.0419	0.0403	0.0386	0.0370
45-49	0.0478	0.0525	0.0456	0.0325	0.0383	0.0464	0.0453	0.0442	0.0431	0.0420	0.0409	0.0398
50-54	0.0502	0.0436	0.0353	0.0259	0.0418	0.0465	0.0461	0.0458	0.0455	0.0451	0.0448	0.0445
55-59	0.0493	0.0350	0.0264	0.0204	0.0443	0.0467	0.0473	0.0479	0.0485	0.0492	0.0498	0.0504
60-64	0.0451	0.0271	0.0197	0.0164	0.0452	0.0465	0.0482	0.0498	0.0514	0.0530	0.0547	0.0563
65-69	0.0386	0.0203	0.0150	0.0140	0.0444	0.0452	0.0478	0.0505	0.0531	0.0557	0.0583	0.0609
70-74	0.0311	0.0147	0.0120	0.0130	0.0418	0.0424	0.0458	0.0492	0.0526	0.0560	0.0594	0.0628
75-79	0.0233	0.0103	0.0098	0.0130	0.0376	0.0378	0.0418	0.0457	0.0496	0.0536	0.0575	0.0614
80-84	0.0161	0.0071	0.0081	0.0134	0.0322	0.0319	0.0360	0.0402	0.0443	0.0485	0.0526	0.0568
85-89	0.0097	0.0050	0.0068	0.0136	0.0261	0.0253	0.0293	0.0334	0.0375	0.0415	0.0456	0.0497
90-94	0.0039	0.0037	0.0058	0.0133	0.0198	0.0186	0.0224	0.0261	0.0298	0.0336	0.0373	0.0411
95-99	-0.0012	0.0028	0.0049	0.0126	0.0138	0.0123	0.0156	0.0188	0.0221	0.0253	0.0285	0.0318
100-104	-0.0054	0.0022	0.0042	0.0115	0.0081	0.0066	0.0092	0.0119	0.0145	0.0171	0.0197	0.0223
105-109	-0.0090	0.0017	0.0036	0.0100	0.0028	0.0014	0.0033	0.0053	0.0072	0.0092	0.0111	0.0131
110+	-0.0122	0.0013	0.0030	0.0084	-0.0022	-0.0036	-0.0023	-0.0011	0.0002	0.0014	0.0027	0.0039

Females

age	e50-55	e55-60	e60-65	e65-70	e70-75	e75-80	e80-85	e85-90	e90-95	e95-100	e100-105	e105-110
0	0.1133	0.1024	0.0878	0.0827	0.0813	0.1025	0.0775	0.0737	0.0698	0.0659	0.0620	0.0581
1-4	0.1004	0.0966	0.0876	0.0875	0.0819	0.0884	0.0732	0.0697	0.0662	0.0627	0.0592	0.0556
5-9	0.0865	0.0904	0.0860	0.0911	0.0821	0.0745	0.0684	0.0652	0.0621	0.0589	0.0558	0.0526
10-14	0.0725	0.0844	0.0829	0.0928	0.0819	0.0616	0.0631	0.0603	0.0575	0.0546	0.0518	0.0490
15-19	0.0607	0.0788	0.0787	0.0924	0.0812	0.0511	0.0579	0.0553	0.0526	0.0500	0.0474	0.0448
20-24	0.0533	0.0739	0.0740	0.0891	0.0795	0.0439	0.0528	0.0503	0.0478	0.0453	0.0429	0.0404
25-29	0.0501	0.0694	0.0696	0.0828	0.0755	0.0400	0.0478	0.0454	0.0430	0.0405	0.0381	0.0356
30-34	0.0499	0.0645	0.0652	0.0738	0.0687	0.0386	0.0431	0.0407	0.0382	0.0358	0.0334	0.0309
35-39	0.0506	0.0587	0.0605	0.0633	0.0600	0.0386	0.0391	0.0367	0.0344	0.0320	0.0297	0.0273
40-44	0.0510	0.0521	0.0550	0.0524	0.0508	0.0393	0.0364	0.0343	0.0322	0.0301	0.0279	0.0258
45-49	0.0507	0.0452	0.0489	0.0422	0.0425	0.0403	0.0356	0.0339	0.0322	0.0305	0.0288	0.0272
50-54	0.0496	0.0384	0.0428	0.0336	0.0358	0.0413	0.0365	0.0355	0.0345	0.0334	0.0324	0.0314
55-59	0.0476	0.0322	0.0368	0.0265	0.0310	0.0423	0.0388	0.0387	0.0385	0.0384	0.0382	0.0381
60-64	0.0442	0.0269	0.0314	0.0210	0.0277	0.0430	0.0419	0.0427	0.0435	0.0443	0.0452	0.0460
65-69	0.0395	0.0224	0.0264	0.0168	0.0251	0.0432	0.0445	0.0463	0.0481	0.0500	0.0518	0.0536
70-74	0.0336	0.0184	0.0218	0.0136	0.0227	0.0423	0.0457	0.0484	0.0511	0.0538	0.0565	0.0592
75-79	0.0269	0.0149	0.0173	0.0110	0.0200	0.0400	0.0448	0.0481	0.0515	0.0549	0.0582	0.0616
80-84	0.0199	0.0116	0.0131	0.0089	0.0169	0.0361	0.0414	0.0452	0.0490	0.0528	0.0565	0.0603
85-89	0.0130	0.0087	0.0092	0.0070	0.0136	0.0309	0.0360	0.0399	0.0439	0.0478	0.0517	0.0556
90-94	0.0066	0.0061	0.0058	0.0053	0.0102	0.0249	0.0293	0.0331	0.0369	0.0407	0.0445	0.0482
95-99	0.0011	0.0039	0.0030	0.0037	0.0070	0.0186	0.0221	0.0255	0.0290	0.0324	0.0359	0.0393
100-104	-0.0034	0.0018	0.0007	0.0023	0.0041	0.0123	0.0149	0.0179	0.0208	0.0238	0.0267	0.0297
105-109	-0.0071	0.0000	-0.0013	0.0009	0.0014	0.0062	0.0079	0.0103	0.0127	0.0150	0.0174	0.0198
110+	-0.0105	-0.0018	-0.0032	-0.0005	-0.0011	0.0003	0.0011	0.0028	0.0046	0.0064	0.0081	0.0099

Table 2 Example of computations of death rates for 2010-2015, Japan, Females

age	mx(2005)	ρx	mx(2010)
0	0.002422	0.073651	0.001937
1	0.000212	0.069693	0.000171
5	0.000087	0.065244	0.000072
10	0.000072	0.060312	0.000060
15	0.000176	0.055254	0.000149
20	0.000285	0.050300	0.000244
25	0.000327	0.045402	0.000285
30	0.000408	0.040678	0.000361
35	0.000589	0.036732	0.000527
40	0.000867	0.034319	0.000781
45	0.001326	0.033888	0.001197
50	0.002013	0.035471	0.001808
55	0.002908	0.038682	0.002586
60	0.004096	0.042695	0.003598
65	0.006234	0.046337	0.005417
70	0.010660	0.048422	0.009204
75	0.019221	0.048139	0.016611
80	0.037066	0.045211	0.032318
85	0.072250	0.039947	0.064008
90	0.133752	0.033106	0.120977
95	0.230339	0.025550	0.213169
100	0.366673	0.017856	0.347349
105	0.528198	0.010281	0.511987
110	0.661972	0.002831	0.656314

mx(2005) – death rates for 2005-2010, life expectancy at birth is 85.95
ρx – pattern of mortality improvement, life expectancy level 85-90, females
mx(2010) – death rates for 2010-2015, life expectancy at birth is 87.12

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